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General Certificate of Education (A-level)
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Mathematics MS03

(Specification 6360)

Statistics 3

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1(a)	r = **0.665 to 0.666** r = **0.6 to 0.7**	B2 (B1)	2	AWFW (0.66577) AWFW
	Alternative Use of a correct formula	(M1)		$\Sigma x = 20.40$ $\Sigma x^2 = 34.785$ $\Sigma xy =$ **49.1155** $\Sigma y = 28.86$ $\Sigma y^2 = 69.4698$
	r = **0.665 to 0.666**	(A1)		$S_{xx} = 0.105$ $S_{yy} = 0.0615$ $S_{xy} =$ **0.0535**
(b)				
	H_0: $\rho = 0$ H_1: $\rho > 0$	B1		Both; do not accept in terms of r but accept in words
	SL $\alpha = 0.01$ (1%)			
	CV r = (+)**0.658**	B1		AWRT (0.6581) H_1: $\rho \neq 0 \Rightarrow$ CV = (±)0.7079
	Calculated $r = 0.666 >$ Tabulated r	M1		Comparison
	Evidence, at 1% level, **of a positive correlation** between the neck length and the tail length of mature male giraffes	A1F	4	F on r and CV At 1% level, **reject hypothesis of no correlation**
	Total		**6**	
2(a)	H_0: $\mu_F = \mu_M$ H_1: $\mu_F \neq \mu_M$	B1		Both; allow suffices of 1 & 2 or X & Y
	SL $\alpha = 0.01$ (1%)			
	CV z = **2.57 to 2.58**	B1		AWFW (2.5758)
	$z = \dfrac{\bar{f} - \bar{m}}{\sqrt{\dfrac{s_F^2}{n_F} + \dfrac{s_M^2}{n_M}}} = \dfrac{22.0 - 21.6}{\sqrt{\dfrac{1.31^2}{50} + \dfrac{0.702^2}{75}}}$	M1 M1		Numerator Denominator; allow ÷49 and ÷74 but **not** pooling
	1.95 to 2(.0)	A1		AWFW (1.98)
	No evidence, at 1% level, to suggest that **there is a difference** between mean lengths	A1F	6	F on CV and z-value allow following pooling
(b)				
	Diameter, thickness, girth, width, weight, shape, colour, texture	B1	1	Accept any sensible alternative but **not** 'quality'
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$P(S \cap U) = 0.15 \times 0.10 = \mathbf{0.015}$	B1	1	CAO
(ii)	$P(O \cap \geq 2) = (0.40 \times 0.50) + (0.45 \times 0.40) + (0.15 \times 0.70)$	M1		≥1 term correct; may be implied
	$= 0.20 + 0.18 + 0.105 = \mathbf{0.485}$	A1	2	CAO
(iii)	$P(U) = (0.40 \times 0.15) + (0.45 \times 0.05) + (0.15 \times 0.10)$ or (i)	M1		≥2 terms correct; may be implied
	$= 0.06 + 0.0225 + 0.015 = \mathbf{0.097 \text{ to } 0.098}$	A1	2	AWFW (0.0975)
(iv)	$P(D \mid U) = \frac{P(D \cap U)}{P(U)} = \frac{0.40 \times 0.15}{(iii)}$	M1		May be implied
	$= \frac{0.06}{0.0975} = \mathbf{0.612 \text{ to } 0.619}$	A1	2	AWFW (0.61538)
(v)	$P(S \mid O) = \frac{0.15 \times (1 - 0.10)}{1 - (iii)} = \frac{0.135}{0.9025}$	M1 M1		Numerator Denominator
	$= \mathbf{0.149 \text{ to } 0.15}$	A1	3	AWFW (0.14958)
(b)	$P(D \cap T \cap S \mid O)$ $= \frac{0.40 \times 0.85}{1 - (iii)} \times \frac{0.45 \times 0.95}{1 - (iii)} \times (v) \times 3!$	M1 M1 M1		≥2 terms correct in numerator (1 – (iii)) in denominator 3! or 6 or 3
	$= \frac{0.34 \times 0.4275 \times 0.135 \times 6}{0.9025^3}$ **or** $= 0.3767 \times 0.4737 \times 0.1496 \times 6$			
	$= \mathbf{0.16}$	A1	4	AWRT (0.16016)
	Total		**14**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	H_0: $\lambda = 2.6$ (650) H_1: $\lambda > 2.6$ (650)	B1		Both; accept μ instead of λ
	SL $\alpha = 0.05$ (5%)			
	CV $z =$ **1.64 to 1.65**	B1		AWFW (1.6449) Allow 1.96 iff H_1: $\lambda \neq 2.6$
	$\hat{\lambda} = \frac{688}{250} =$ **2.75(2)**	B1		AWRT (2.752) Can be implied by use of 688
	$z = \frac{2.752 - 2.6}{\sqrt{\frac{2.6}{250}}} = \frac{688 - 650}{\sqrt{650}} =$ **1.47 to 1.49**	M1 A1		Allow use of 2.752 or 688 or 687.5 in denominator AWFW
	p-value = **0.068 to 0.071** > 0.05	(M1)		Use of $P(X \geq 688 \mid \lambda = 650) = 0.072$ $\Rightarrow$ M0 A0 (M1) AF1
	No evidence, at 5% level, **to support** manager's suspicion	A1F	6	F on CV and z-value
	Total		**6**	
5(a)	$\hat{p} = \frac{68}{125} =$ **0.544**	B1		CAO; or equivalent
	98% $\Rightarrow$ $z =$ **2.32 to 2.33**	B1		AWFW (2.3263)
	Approximate CI for p is $0.544 \pm 2.3263 \times \sqrt{\frac{0.544 \times 0.456}{125}}$	M1 A1F		F on $\hat{p}$ and z
	0.544 ± (0.103 to 0.104) or **(0.44 to 0.441, 0.647 to 0.65)**	A1		CAO/AWFW May be implied by **correct** answer AWFW
	(44%, 65%)	A1	6	AWRT
(b)	Require $2 \times 2.3263 \times \sqrt{\frac{p(1-p)}{n}} \leq 0.1$ (10%)	M1 A1		Allow 'no 2' and FT on CI from (a) Allow $p = 0.44$ to 0.65
	Thus $\sqrt{n} = 22.1$ to 23.3	M1		Attempted solution for $\sqrt{n}$ or n
	***n* = 490, 495, ..., 545**	A1	4	Must be to 'nearest 5'
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$M = U + V$			
	$E(M) = 13 + 15 = \mathbf{28}$	B1		CAO
	$V(M) = 3^2 + 6^2$	M1		
	$+ 2 \times 3 \times 6 \times (-0.6)$	M1		Allow 'no 2'
	$= 9 + 36 - 21.6 = \mathbf{23.4}$	A1	4	CAO
(ii)	$D = W - 2U$			
	$E(D) = 24 - 2 \times 13 = \mathbf{-2}$	B1		CAO
	$V(D) = 4^2 + (2^2 \times 3^2)$	M1		
	$= 16 + 36 = \mathbf{52}$	A1	3	CAO
(iii)	$T = M + W + X$			
	$E(T) = 28 + 24 + 9 = \mathbf{61}$	B1F		F on E(M) from (a)(i)
	$V(T) = 23.4 + 4^2 + 2^2 = \mathbf{43.4}$	B1F	2	F on V(M) from (a)(i)
(b)(i)	$P(M = 30) = \mathbf{0}$	B1	1	CAO
(ii)	$P(W > 2U) = P(D > 0) = P\left(Z > \frac{0-(-2)}{\sqrt{52}}\right)$	M1		Standardising 0 using c's E(D) & c's V(D) from (a)(ii)
	$= P(Z > 0.28) = 1 - 0.61026$	m1		Area change
	$= \mathbf{0.389 \text{ to } 0.394}$	A1	3	AWFW (0.39076)
(iii)	$P(50 < T < 70) =$			
	$P\left(Z < \frac{70-61}{\sqrt{43.4}}\right) - P\left(Z < \frac{50-61}{\sqrt{43.4}}\right)$	M1		Standardising 70 & 50 using c's E(T) & c's V(T) from (a)(iii)
	$= P(Z < \mathbf{1.37}) - P(Z < \mathbf{-1.67})$	A1		AWRT either
	$= 0.91466 - (1 - 0.95254)$	m1		Area change
	$= \mathbf{0.865 \text{ to } 0.87}$	A1	4	AWFW (0.86657)
	Total		**17**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$X \sim \text{B}(n,\ p)$			
	$\text{E}(X) = \sum_{x=0}^{n} x \times \binom{n}{x} p^x (1-p)^{n-x}$	M1		Used; ignore limits until A1
	$= np \times \sum_{x=1}^{n-1} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1}(1-p)^{n-x}$	M1		Factor of np plus p^x to p^{x-1}, $n!$ to $(n-1)!$ and $x!$ to $(x-1)!$ Ignore limits until A1
	$= np \times \sum_{x=1}^{n-1} \text{B}(n-1,\ p) = \boldsymbol{np}$	A1	3	Fully correct and complete derivation AG
(ii)	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$			
	$= \text{E}[X(X-1)] + \text{E}(X) - [\text{E}(X)]^2$	M1		Used (Other derivations are possible)
	$= n(n-1)p^2 + np - n^2p^2 = \boldsymbol{np(1-p)}$	A1	2	Fully correct and complete derivation
(b)(i)	$\binom{n}{m} p^m (1-p)^{n-m} \geq \binom{n}{m-1} p^{m-1}(1-p)^{n-m+1}$	M1		Stated or used
	$\frac{n!}{m!(n-m)!} p^{n-m} \geq \frac{n!}{(m-1)!(n-m+1)!}(1-p)^{n-m+1}$	m1		Expanding $\binom{n}{m}$ and $\binom{n}{m-1}$
	$(n-m+1)\,p \geq m(1-p)$ $np - mp + p \geq m - mp$	A1		Simplification
	$\boldsymbol{m \leq (n+1)p}$	A1	4	Fully correct and complete derivation AG
(ii)	B(10, 0.65)			
	$(n+1)p = 7.15$ and $(n+1)p - 1 = 6.15$	M1		Can be implied by '7'
	$m = \mathbf{7}$	A1		CAO
	B(35, 0.5)			
	$(n+1)p = 18$ and $(n+1)p - 1 = 17$	(M1)		Can be implied by '17 or 18'
	$m =$ **17 and 18**	A1	3	CAO; accept 'or'
(c)	$Y \sim \text{B}(4000,\ 0.00095) \approx$ **Po(3.8)**	B1		May be implied
	$\boldsymbol{k = 3}$	B1		CAO; may be implied
	$\text{P}(Y \leq 3) =$ **0.473 to 0.474**	B1	3	AWFW (0.4735)
	SC: $\boldsymbol{k = 4} \Rightarrow \text{P}(Y \leq 4) =$ **0.667 to 0.668** Award B1 B0 B1			AWFW (0.6678)
	Total		**15**	
	TOTAL		**75**	